N_{Re} = Reynolds number, $U_w \delta_s / \nu$
N_{Sc} = Schmidt number, ν / D
T = temperature, $T = \overline{T} + \theta$
T_o = arbitrary reference temperature
t = time
$\tilde{t}$ = $t\, U_w / \delta_s$, dimensionless time
U_i = velocity in direction i, $U_i = \overline{U_i} + u_i$
U_w = wave velocity, β / k_x
u', v', w' = root-mean square velocity fluctuations in x, y, and z directions
$\tilde{u}_k$ = u_k / U_w, dimensionless velocity fluctuation in direction k.
$\hat{v}$ = $f_2(y)/U_w$, dimensionless amplitude of the normal velocity fluctuation
x = distance parallel to the wall in the direction of the mean flow
$\tilde{x}$ = x / δ_s
y = distance normal to the wall
y^+ = $u^* y / \nu$
Y = $\sqrt{\tilde{\beta}/2\nu}\, y$

Greek Letters

β = frequency, real
$\tilde{\beta}$ = $\beta \delta_s / U_w$
δ_s = sublayer thickness
ϵ_H = eddy diffusivity defined by Equation (1)
η = stretched variable defined by Equation (13)
θ = fluctuating part of the temperature
$\tilde{\theta}$ = θ / T_o, dimensionless temperature fluctuations
$\hat{\theta}$ = dimensional amplitude of the temperature fluctuations
ν = kinematic viscosity
ζ, ξ = phases, functions of y
λ = $(N_{Re}\, \beta / 2)^{1/2}$

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Concerning the Calculation of Residence Time Distribution Functions for Systems in Which Diffusion is Negligible

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In order to predict the chemical performance of a reactor, a residence time distribution function must be known (*1*). The F curve is particularly convenient for such calculations. However, for reasons of economy, inlet changes in tracer concentration approximately a delta function are often used in experimental studies; for convenience the f curve is defined as the fraction of dye with residence time t or less.

When the characteristic time associated with diffusion normal to the streamlines is small compared to the characteristic residence time, the analytical method presented by Taylor (*2*) and extended by Aries (*3*), Saffman (*4*), and Lighthill (*5*) may be used. When both characteristic times are nearly equal, then a numerical solution similar to that of Farrell and Leonard (*6*) must be obtained. When the characteristic time associated with diffusion is large compared to the characteristic residence time, the method presented by Bosworth (*7, 8*) and noted by Denbigh (*9*) is applicable. However, the method of Bosworth is somewhat cumbersome and usually requires a relatively involved integration. Presented below is a calculation scheme based upon physical reasoning which requires much less work than Bosworth's method when the velocity profile can be solved explicitly for the distance coordinate. In addition, this formulation aids in understanding the relationship between the f curve and the F curve. When the velocity profile cannot be solved explicitly for the distance coordinate, both methods require about the same amount of work.

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As an example of this method, we now consider the case of steady Couette flow as shown in Figure 1. Thus, we have the velocity profile

$$u = u_0 \left[1 - (y/\delta)\right] \tag{1}$$

where u_0 is the velocity at $y = 0$. The distance, x, that a fluid element travels in the time interval t is

$$x = u_0 t \left[1 - (y/\delta)\right] \tag{2}$$

In this analysis we shall consider residence time distribution in the system between the sections $x = 0$ and $x = L$.

DELTA FUNCTION

Recall the experiments reported by Goldish (*10*) and by Koutsky and Adler (*11*); that is, the fluid is consid-

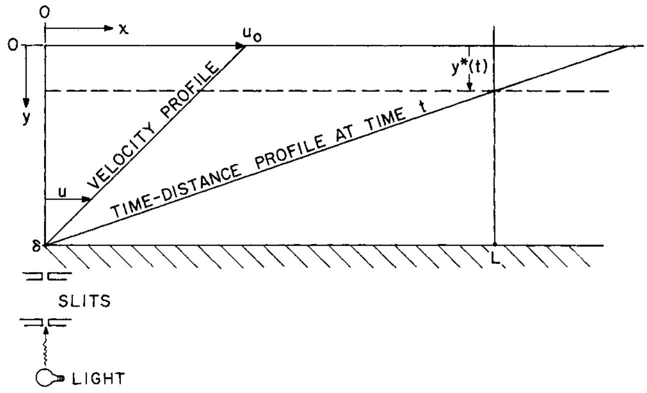

Fig. 1.

Table 1.

Case	U/U_o	Distribution Functions
1. Couette flow	$1-y/\delta$	$f(t)=1-(t_o/t)$ $F(t)=2f(t)-f^2(t)$
2. Falling film flow	$1-(y/\delta)^2$	$f(t)=[1-(t_o/t)]^{1/2}$ $F(t)=\frac{3}{2}f(t)-\frac{1}{2}f^3(t)$
3. Laminar film flow	$\beta(1-y/\delta)$ + $(1-\beta)[1-(y/\delta)^2]$ where $o\leq\beta\leq 1$	$f(t)=\frac{-\beta+[\beta^2+4(1-\beta)(1-t_o/t)]^{1/2}}{2(1-\beta)}$ $F(t)=\left(\frac{6}{4-\beta}\right)f(t)-\left(\frac{3\beta}{4-\beta}\right)f^2(t)+\frac{2(\beta-1)}{4-\beta}f^3(t)$
4. Poiseuille flow	$1-(r/R)^2$	$f(t)=1-(t_o/t)$ $F(t)=2f(t)-f^2(t)$
5. Turbulent film flow	$[1-(y/\delta)]^{1/n}$ where $n>5$	$f(t)=1-(t_o/t)^{-n}$ $F(t)=1-(t_o/t)^{-(n+1)}$
6. Turbulent tube flow	$[1-(r/R)]^{1/n}$ where $n>5$	$f(t)=[1-(t_o/t)^n]^2$ $F(t)=1+\frac{(1+n)(t_o/t)^{1+2n}-(1+2n)(t_o/t)^{1+n}}{n}$

ered to be a dilute solution of a solute which ordinarily is colorless, but irreversibly and completely reacts instantly to form a colored solute when exposed to a light source as shown in Figure 1. At $t=0$, the light is turned on for a very short time interval. At $t>0$, the colored solute will form a line given by Equation (2). At $t=t_0=L/u_0$, the first trace of colored solute will reach $x=L$, and thereafter dye will be crossing the $x=L$ plane. At $t>t_0$, the dye will cross the $x=L$ plane at $y=y^*$, which from Equation (2) is $y^*=\delta[1-(L/u_0t)]$. Thus the fraction, $f(t)$, of the dye which has a residence time t or less is

$$f(t)=\int_0^{y^*(t)}dy\bigg/\int_0^{\delta}dy=1-t_0/t \qquad (3)$$

where $t>t_0$.

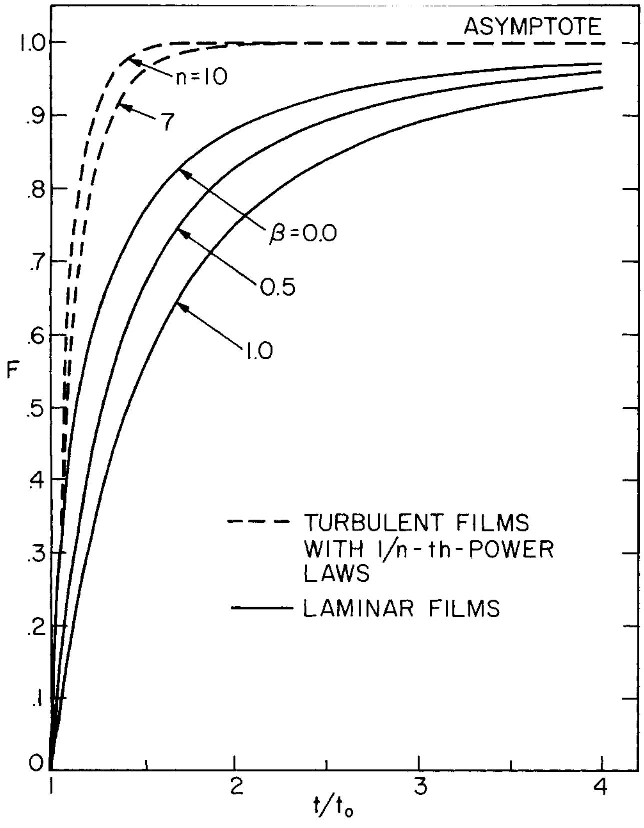

Fig. 2.

STEP FUNCTION

In this case the light is turned on at $t=0$ and left on throughout the experiment. In this case the fraction, $F(t)$, of the dye which has a residence time t or less is given by

$$F(t)=\int_0^{y^*(t)}u\,dy\bigg/\int_0^{\delta}u\,dy=1-(t_0/t)^2 \qquad (4)$$

Results of similar calculations are presented in Table 1. The first two cases represent limiting solutions for case 3. It can be shown that the expressions of $F(t)$ presented for cases 4 and 6 are identical to those reported elsewhere (7, 8). Although cases 5 and 6 are seldom applicable to practical situations because diffusion is neglected, they have been presented in order to provide a comparison with the corresponding laminar systems. Typical results are shown in Figure 2.

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